Learning Target(s): I am able to find angles given values of trigonometric functions.
I am able to find inverse trig functions in both radians and degrees for sine, cosine, and tangent.

13.4 Notes: Evaluate Inverse Trigonometric Functions

inverse

Definition: used to ~~find~~ find an angle

Symbols: $\sin^{-1}$ or arcsin

Calculator: $\boxed{2^{nd}}$ $\boxed{\sin}$

Ex. 1
Use a calculator to evaluate the expression in both radians and degrees.

a. $\tan^{-1}(21.1)$

87.3°
1.5 rad

b. $\sin^{-1}(-0.78)$

−51.3°
−0.9 rad

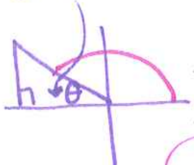
c. $\arccos(0.54)$

Ex. 2
Solve the equation for θ.

a. sin θ = 0.41; 90° < θ < 180°

$\sin^{-1}(0.41)$ (degrees)

24.2°

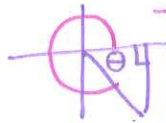


$$\begin{array}{r} 180 \\ -24.2 \\ \hline 155.8° \end{array}$$

b. cos θ = 0.3; 270° < θ < 360°

$\cos^{-1}(0.3)$

72.5°

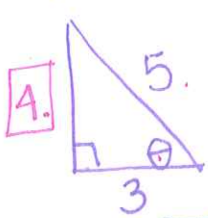


$$\begin{array}{r} 360 \\ -72.5 \\ \hline 287.5° \end{array}$$

c. tan θ = 8.2; 180° < θ < 270°

Ex. 3
Find the exact value of expression (not a special unit circle value):

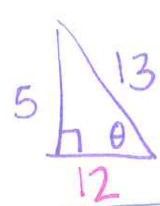
a.) $\sin\left(cos^{-1}\frac{3}{5}\right)$ cos ratio



Find missing side using triples or Pyth. Thm

$$\boxed{\sin\theta = \frac{4}{5}}$$

b.) $\tan\left(sin^{-1}\frac{5}{13}\right)$



$$\boxed{\tan\theta = \frac{5}{12}}$$

$y = \sin x$

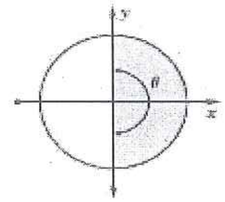
D: $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

R: $-1 \le y \le 1$

$y = \sin^{-1}x$ ($y = \arcsin x$)

D: $-1 \le x \le 1$

R: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



$y = \sin^{-1}x$ if and only if $y = \sin x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

$y = \cos x$

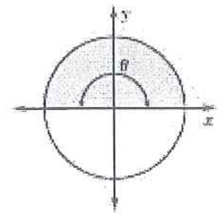
D: $0 \le x \le \pi$

R: $-1 \le y \le 1$

$y = \cos^{-1}x$ ($y = \text{arcscos } x$)

D: $-1 \le x \le 1$

R: $0 \le y \le \pi$



$y = \cos^{-1}x$ if and only if $y = \cos x$ and $0 \le y \le \pi$.

$y = \tan x$

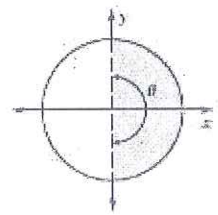
D: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

R: All Real Numbers

$y = \tan^{-1}x$ ($y = \arctan x$)

D: All Real Numbers

R: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



$y = \tan^{-1}x$ if and only if $y = \tan x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

The inverse function means that the domain of the original function has been restricted so the inverse will be a function. These restricted values are called *Principal Values.*

Ex. 4 Evaluate the expression without using a calculator. Give your answer in both radians and degrees.

a. Find $\arccos \frac{\sqrt{3}}{2}$. $30°, \frac{\pi}{6}$

*Ask yourself: *What angle has a cos equal to* $\frac{\sqrt{3}}{2}$?

b. Find $\sin^{-1} -\frac{\sqrt{2}}{2}$. $315°, \frac{7\pi}{4}$

Ask yourself: What angle has a sin equal to $-\frac{\sqrt{2}}{2}$?

Try it!

Find arctan (-1). $(-45°)$ $315°, \frac{7\pi}{4}$

Ask yourself: What angle has a tan equal to -1?

Challenge:

Find the exact value of each expression:

a.) $\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)$

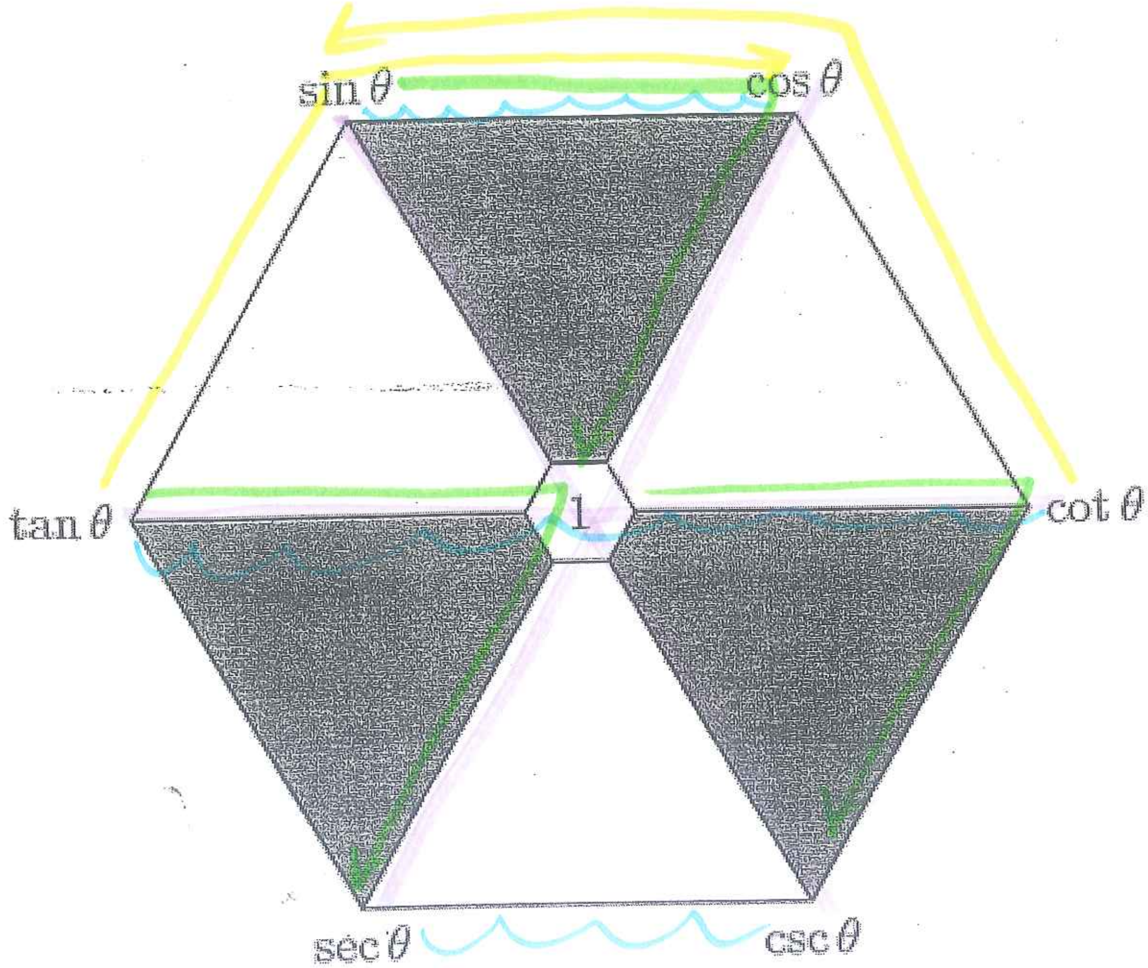
b.) $\cos\left(\sin^{-1} -\frac{\sqrt{2}}{2}\right)$

c.) $\cos\left(\tan^{-1}\sqrt{3}\right)$

d.) $\arcsin\left(\sin\frac{7\pi}{4}\right)$

e.) $\arccos\left(\cos\frac{7\pi}{6}\right)$

f.) $\tan^{-1}\left(\cos\frac{3\pi}{2}\right)$



Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Learning Target(s): I am able to simplify a trigonometric expression.
I am able to verify trigonometric identities using a variety of techniques.
I am able to apply the following identities: Reciprocal, Tangent/Cotangent, Pythagorean, and Co-function.

14.3 Verify Trigonometric Identities

Trigonometric Identity – A trig equation that is true for all values of the variable θ.

Ex. 1
What is another way to write these identities?

a) $\sin^2\theta - 1$

$$\sin^2\theta + \cos^2\theta = 1$$
$$-1 - \cos^2\theta \quad -1 \quad -\cos^2\theta$$
$$\sin^2\theta - 1 = \boxed{-\cos^2\theta}$$

b) $\sec^2\theta - \tan^2\theta$

$$1 + \tan^2\theta = \sec^2\theta$$
$$-\tan^2\theta \quad\quad -\tan^2\theta$$
$$\boxed{1} = \sec^2\theta - \tan^2\theta$$

Ex. 2
Given that $\sin\theta = \frac{4}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the values of the other 5 trig functions of θ.

$\cos\theta =$ _______

$\tan\theta =$ _______

$\csc\theta =$ _______ $\sec\theta =$ _______ $\cot\theta =$ _______

Try it!

1. Given that $\cos\theta = \dfrac{-3}{4}$ and $\pi < \theta < \dfrac{3\pi}{2}$ find the values of the other 5 trig functions of θ.

$\sec\theta =$ _______

$\sin\theta =$ _______ $\csc\theta =$ _______

$\tan\theta =$ _______ $\cot\theta =$ _______

Ex. 3

Simplify the expression $\tan(\frac{\pi}{2} - \theta)\sin\theta$.

$\cot\theta$

$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta} = \boxed{\cos\theta}$

Ex. 4

Simplify the expression $\tan(-\theta)\cos\theta$.

$-\tan\theta\cos\theta$

$-\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}$

$\boxed{-\sin\theta}$

Ex. 5

Simplify the expression $\csc\theta\cot^2\theta + \frac{1}{\sin\theta}$

$= \csc\theta(\csc^2\theta - 1) + \csc\theta$

$= \csc^3\theta - \cancel{\csc\theta} + \cancel{\csc\theta}$

$= \csc^3\theta$

$\frac{1}{\sin\theta} \cdot \csc^2\theta - 1 + \frac{1}{\sin\theta}$

$\frac{\csc^2\theta - 1}{\sin\theta} + \frac{1}{\sin\theta}$

$\frac{\csc^2\theta \cancel{-1} \cancel{+1}}{\sin\theta} = \csc^2\theta \cdot \csc\theta$

$= \csc^3\theta$

Ex. 6

Simplify the expression $\sin\theta + \cos\theta\cot\theta$

$\sin\theta \cdot \sin\theta + \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \frac{1}{\sin\theta} = \boxed{\csc\theta}$

Try it!

2. Simplify the expression $\sin x \cot x \sec x$.

$\cancel{\sin x} \cdot \frac{\cancel{\cos x}}{\cancel{\sin x}} \cdot \frac{1}{\cancel{\cos x}}$

$= \boxed{1}$

3. Simplify the expression $\frac{\tan x \csc x}{\sec x}$.

$\frac{\frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \frac{\cancel{1}}{\cancel{\sin x}}}{\frac{\cancel{1}}{\cancel{\cos x}}} = \boxed{1}$

4. Simplify the expression $\frac{\cos(\frac{\pi}{2} - \theta) - 1}{1 + \sin(-\theta)}$

$\frac{\sin\theta - 1}{1 - \sin\theta}$

$\frac{-(\cancel{-\sin\theta + 1})}{\cancel{1 - \sin\theta}} = \boxed{-1}$

5. Simplify the expression: $\dfrac{1}{\sin(\frac{\pi}{2}-\theta)}\cdot\cot\theta$

$\dfrac{1}{\cancel{\cos\theta}}\cdot\dfrac{\cancel{\cos\theta}}{\sin\theta}$

$\dfrac{1}{\sin\theta}$

$\boxed{\csc\theta}$

Challenge:

Simplify the expression: $\dfrac{\tan\theta}{\sec\theta}\cdot\sin\theta+\tan\theta\cdot\csc\theta\cdot\cos^3\theta$

$\dfrac{\sin\theta}{\cancel{\cos\theta}}\cdot\dfrac{\cancel{\cos\theta}}{1}\cdot\sin\theta+\dfrac{\cancel{\sin\theta}}{\cos\theta}\cdot\dfrac{1}{\cancel{\sin\theta}}\cdot\cos^3\theta$

$\sin^2\theta+\dfrac{\cos^3\theta}{\cos\theta}$

$\sin^2\theta+\cos^2\theta$

$\boxed{1}$

Ex. 7
Verify the identity $\sec\theta-\cos\theta=\sin\theta\tan\theta$.

$\dfrac{1}{\cos\theta}-\cos\theta$ $\qquad$ $\sin\theta\tan\theta$ ✓

$\dfrac{1-\cos^2\theta}{\cos\theta}$

$\dfrac{\sin^2\theta}{\cos\theta}$

$\sin\theta\cdot\dfrac{\sin\theta}{\cos\theta}$

*Pick more "difficult" looking side & break it down

* can only work on 1 side

* Multiply by a denominator to get same denominator

Ex. 8
Verify the identity $\sec x+\tan x=\frac{\cos x}{1-\sin x}$.

$\dfrac{1}{\cos x}+\dfrac{\sin x}{\cos x}$

$\dfrac{1+\sin x}{\cos x}\cdot\dfrac{1-\sin x}{1-\sin x}$

$\dfrac{1-\sin^2 x}{\cos x(1-\sin x)}$

$\dfrac{\cos^{\cancel{2}}x}{\cancel{\cos x}(1-\sin x)}$

$\dfrac{\cos x}{1-\sin x}$ ✓

Ex. 9
Verify the identity $\sin\theta(\tan\theta+\cot\theta)=\sec\theta$.

$\sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\sin\theta}{\sin\theta}+\dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{\cos\theta}{\cos\theta}\right)$

$\sin\theta\left(\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)$

$\cancel{\sin\theta}\left(\dfrac{1}{\cancel{\sin\theta}\cos\theta}\right)$

$\dfrac{1}{\cos\theta}$

$\sec\theta$ ✓

Try it!

Verify the identities.

6. $\cos x \csc x \tan x = 1$

$\cancel{\cos x} \cdot \frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}}$

$1 \checkmark$

7. $\csc^2 x\,(1 - \sin^2 x) = \cot^2 x$

$\frac{1}{\sin^2\theta} \cdot \cos^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta}$

$\cot^2 x \checkmark$

8. $(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$

$\sec^2 x \cdot -\sin^2 x$

$\frac{1}{\cos^2 x} \cdot -\sin^2 x$

$\frac{-\sin^2 x}{\cos^2 x}$

$-\tan^2 x \checkmark$

Challenge:

Verify the identity.

$$\sec\theta \cdot \frac{1}{\cos\theta} - \tan\theta \cdot \cot\theta = \frac{1 - \cos^2\theta}{1 - \sin^2\theta}$$

$\frac{1}{\cos\theta} \cdot \frac{1}{\cos\theta} - \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}$

$\frac{1}{\cos^2\theta} - 1 \cdot \frac{\cos^2\theta}{\cos^2\theta}$

$\frac{1 - \cos^2\theta}{\cos^2\theta}$

$\frac{1 - \cos^2\theta}{1 - \sin^2\theta} \checkmark$