

Learning Target(s): I am able to solve exponential and logarithmic equations.

Notes: 7.6 Solve Exponential and Logarithmic Equations

Exponential equation – an equation in which variable expressions occur as exponents.

$$5^x = 5^4 \quad x=4$$

If same base, equate the exponents.

Ex 1: Solve  $64^x = 16^{x+1}$

$$2^{6x} = 2^{4(x+1)}$$

$$2^{6x} = 2^{4x+4}$$

$$6x = 4x + 4$$

$$-4x = -4x$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

Not the same base, so must REWRITE to make the same base.

Ex 2: Solve  $6^x = 27$

~~$$\log_6 6^x = \log_6 27$$~~

$$x = \log_6 27$$

$$x = \frac{\log 27}{\log 6} \approx 1.84$$

Not the same base, AND you CANNOT REWRITE to make the same base. So...MUST LOG each side. (Use log of base that has the variable exponent)

change of base (7.5)

$$\boxed{1.84}$$

Ex 3: Solve  $6e^{0.25x} + 8 = 20$

$$\frac{6e^{0.25x}}{6} = \frac{12}{6}$$

~~$$\log_e e^{0.25x} = \log_e 2$$~~

$$\frac{0.25x}{0.25} = \frac{\ln 2}{0.25}$$

Not the same base, AND you CANNOT REWRITE to make the same base. So...MUST LOG each side. (Use log of base that has the variable exponent)

\*\*Isolate the variable first!!!

$$\log_e = \ln$$

$$\boxed{x \approx 2.77}$$

Try it! Solve the equation.

1.  $3^{7x-3} = 9^{2x}$

2.  $8^{3x+2} - 6 = 5$

3.  $3e^{0.5x} + 2 = 5$

$$x=1$$

$$x = -0.282$$

$$x=0$$

**Logarithmic equation** – An equation that involves logarithms of variable expressions.

Ex 4: Solve  $\log_7(6x-16) = \log_7(x-1)$

If same base, equate the "answers".

$$\begin{array}{r} 6x-16 = x-1 \\ -x+16 \quad -x+16 \\ \hline 5x = 15 \\ \underline{5} \quad \underline{5} \\ x = 3 \end{array}$$

$$x = 3$$

$$\begin{array}{l} \checkmark \log_7(6 \cdot 3 - 16) \stackrel{?}{=} \log_7(3-1) \\ \log_7(2) \stackrel{?}{=} \log_7(2) \checkmark \end{array}$$

**\*\*Note:** You must check your answer in the original problem. You cannot take the log of a negative #.

Ex 5: Solve  $\log_5(3x-8) = 2$

$$\log_B A = E$$

Not a log on both sides, so MUST REWRITE in EXPONENTIAL form.

$$\begin{array}{r} 5^2 = 3x-8 \\ 25 = 3x-8 \\ +8 \quad +8 \\ \hline 33 = 3x \\ \underline{3} \quad \underline{3} \\ x = 11 \end{array}$$

$$x = 11$$

$$\begin{array}{l} \checkmark \log_5(3 \cdot 11 - 8) \stackrel{?}{=} 2 \\ \log_5(33-8) \stackrel{?}{=} 2 \\ \log_5(25) = 2 \checkmark \end{array}$$

**\*\*Remember always check your answers in the original equation.**

Ex 6: Solve  $\log 5x + \log(x-1) = 2$

Not a log on both sides, so MUST REWRITE in EXPONENTIAL form.

**\*\*Condense the logs into ONE LOG first!!!**

(Use 7.5 Foldable for properties)

$$\log 5x(x-1) = 2$$

$$\log(5x^2 - 5x) = 2$$

$$10^2 = 5x^2 - 5x$$

$$100 = 5x^2 - 5x$$

$$\begin{array}{r} -100 \\ \hline 0 = \frac{5x^2}{5} - \frac{5x}{5} - \frac{100}{5} \\ 0 = x^2 - x - 20 \end{array}$$

$$0 = (x-5)(x+4)$$

$$\begin{array}{r} 0 = x-5 \\ +5 \quad +5 \\ \hline 5 = x \end{array} \quad \begin{array}{r} 0 = x+4 \\ -4 \quad -4 \\ \hline -4 = x \end{array}$$

$$\begin{array}{l} \log 5(5) + \log(5-1) \stackrel{?}{=} 2 \\ \log 25 + \log 4 \stackrel{?}{=} 2 \\ \log 100 = 2 \checkmark \\ \log 5(-4) + \log(-4-1) \stackrel{?}{=} 2 \\ \log -20 + \log -5 \stackrel{?}{=} 2 \end{array}$$

**\*\*Don't forget to check your answers!**

Try it!

4.  $\ln(7x-13) = \ln(2x+17)$

5.  $\log_4(10x+624) = 5$

6.  $\log_6(x-9) + \log_6 x = 2$

$$x = 6$$

$$x = 40$$

$$x = 12, x \neq 3$$