Learning Target(s): I can use the Fundamental Theorem of Algebra to find all zeros of a polynomial function.
I can use the Fundamental Theorem of Algebra to find the number of solutions to a polynomial function.
I can use the Conjugates Theorem to write the equation of a polynomial function given the zeros.

Notes: 5.7 Apply the Fundamental Theorem of Algebra

$x^2-4x+4=0$

$(x-2)(x-2)=0$

$x=2$

Repeated Solution – for the equation f(x) = 0, k is a repeated solution if and only if the factor $x-k$ has a degree greater than 1 when f is factored completely.

The Fundamental Theorem of Algebra – if f(x) is a polynomial of degree n where n > 0 then the equation f(x) = 0 has at least one solution.

Corollary – if f(x) is a polynomial of degree n, then the equation f(x) = 0 has exactly n solutions provided each solution repeated twice is counted as 2 solutions and so on.

Ex 1: Find the number of solutions or zeros for each equation or function.

a. $x^3-3x^2+9x-27=0$

deg: 3

3 solutions

b. $x^4+6x^3-32x=0$

deg: 4

4 solutions

Ex 2: Find all zeros of $f(x)=x^4-7x^3+13x^2+x-20$

p: $20 \rightarrow \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

q: $1 \rightarrow \pm1$

$\frac{p}{q}=\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}$

$$\begin{array}{r|rrrrr} -1 & 1 & -7 & 13 & 1 & -20 \\ & & -1 & 8 & -21 & 20 \\ \hline & 1x^3 & -8x^2 & 21x & -20 & \boxed{0} \end{array}$$ ✓

$$\begin{array}{r|rrrr} 4 & 1 & -8 & 21 & -20 \\ & & 4 & -16 & 20 \\ \hline & 1x^2 & -4x & +5 & \boxed{0} \end{array}$$ ✓

$2\pm i$

$\frac{4\pm2i}{2}$

x^2-4x+5

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}=\frac{4\pm\sqrt{-4}}{2}$

$x=-1, 4, 2\pm i$

Complex Conjugates Theorem – If f is a polynomial function with real coefficients and $a+bi$ is an imaginary zero of f, then $a-bi$ is also a zero of f.

$5+i,\ 5-i$

comes from Quadratic Formula

Irrational Conjugates Theorem – if $a+\sqrt{b}$ is a zero of f, then $a-\sqrt{b}$ is also a zero of f.

$3+\sqrt{10}$
$3-\sqrt{10}$

always

Ex 3: Write a polynomial function f of least degree that has real coefficients, a leading coefficient of 1, and -2 and $3+i$ as zeros.

x^3

**Since $3+i$ is a zero then $3-i$ must also be a zero.

Write the product of the factors.

$$f(x) = (x+2)[(x-(3+i))][(x-(3-i))]$$

Regroup

$$[(x-3)+i][(x-3)-i] \quad \text{FOIL}$$

$$(x-3)(x-3)-i^2 \quad i^2=-1$$

$$x^2-6x+9+1$$

$$(x+2)(x^2-6x+10)$$

$$\boxed{f(x) = x^3-4x^2-2x+20}$$

Try it!

1. Write a polynomial of least degree that has rational coefficients, a leading coefficient of 1, and 4 and $1+\sqrt{6}$ as zeros.

$1-\sqrt{6}$

$$f(x) = (x-4)((x-(1+\sqrt{6}))((x-(1-\sqrt{6}))$$

$$(x-1)(x-1)-\sqrt{6}\cdot\sqrt{6}$$

$$(x-1)(x-1)-\sqrt{36}$$

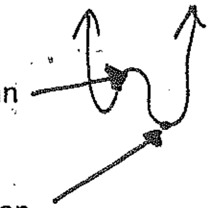
$$x^2-2x+1-6$$

$$(x-4)(x^2-2x-5)$$

$$\boxed{f(x) = x^3-6x^2+3x+20}$$

Learning Target(s): I am able to use x-intercepts to graph a polynomial function.
I am able to identify turning points of a polynomial functions to help graph the function.

Notes: 5.8 Analyze Graphs of Polynomial Functions



Local Maximum – the y-coordinate of a turning point if the point is higher than all the nearby points.

Local Minimum - – the y-coordinate of a turning point if the point is lower than all the nearby points.

Zeros, factors, solutions, and intercepts

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ be a polynomial function. If k is a real number then the following statements are equivalent.

Zero: k is a zero of the polynomial function f.

Factor: x − k is a factor of the polynomial function f(x).

Solution: k is a solution of the polynomial equation f(x) = 0.

x-intercept: k is an x-intercept of the graph of the polynomial function f. The graph of f contains the point (k, 0).

Turning Points of Polynomial Functions

The graph of every polynomial function of degree n has at most n − 1 turning points. If a polynomial function has n distinct real zeros then its graph has exactly n − 1 turning points.

Ex 1: Graph the function.

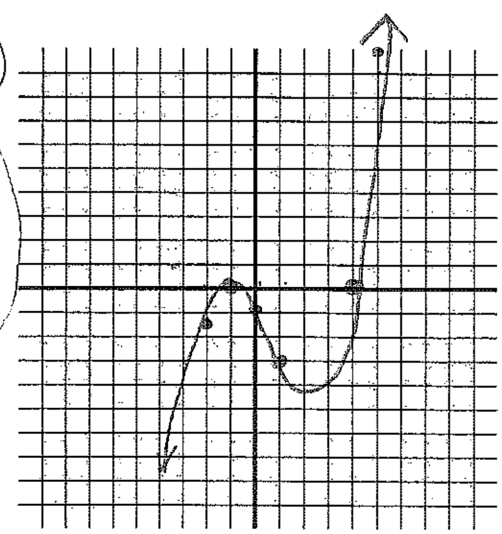
$$f(x) = \frac{1}{4}(x+1)^2(x-4)$$

Step 1: Use the intercepts. (−1, 0), (4, 0)

Step 2: Plot points between and beyond the x-intercepts.

x	y
-2	-1.5
0	-1
1	-3
5	9

show me your table!



Step 3: Check end behavior → if you do not have a graphing calculator.

deg 3 odd l.c +

Step 4: Draw the graph through the plotted points.

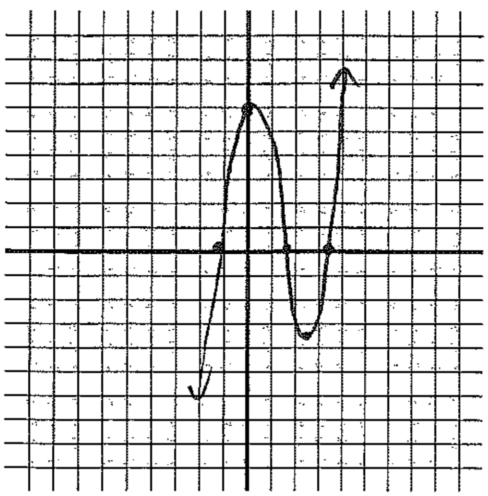
Ex 2: Graph the function. Identify the x-intercepts and the local maximums and local minimums.

a. $f(x) = x^3 - 4x^2 + 6$

x-int(s): (-1.1, 0) (1.6, 0) (3.5, 0)

Max(s): (0, 6)

Min(s): (2.7, -3.5)

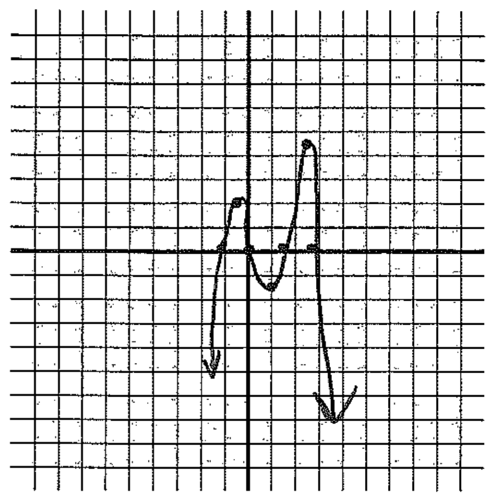


b. $f(x) = -x^4 + 3x^3 + x^2 - 4x$

x-int(s): (-1.1, 0) (0, 0) (1.3, 0) (2.9, 0)

Max(s): (-.7, 2) (2.3, 4.6)

Min(s): (.7, -1.5)

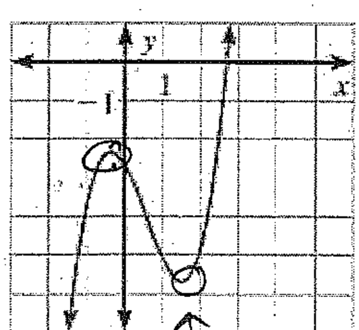


Ex. 3

Estimate the coordinates of each turning point and state whether each corresponds to a local maximum or a local minimum. Then estimate all real zeros and determine the least degree the function can have.

a.

max

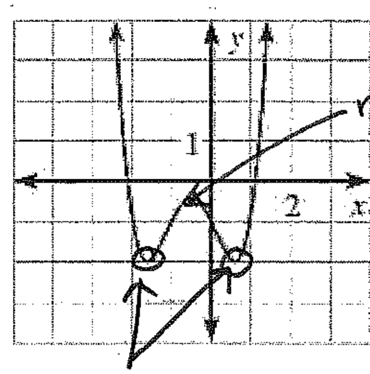


min

2 turning pts

deg 3

b.



max

min

3 turning pts

4 deg