Learning Target(s): I can use the Fundamental Theorem of Algebra to find all zeros of a polynomial function.
I can use the Fundamental Theorem of Algebra to find the number of solutions to a polynomial function.
I can use the Conjugates Theorem to write the equation of a polynomial function given the zeros.

Notes: 5.7 Apply the Fundamental Theorem of Algebra

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

Repeated Solution – for the equation f(x) = 0, k is a repeated solution if and only if the factor x-k has a degree greater than 1 when f is factored completely.

The Fundamental Theorem of Algebra – if f(x) is a polynomial of degree n where n > 0 then the equation f(x) = 0 has at least one solution.

Corollary – if f(x) is a polynomial of degree n, then the equation f(x) = 0 has exactly n solutions provided each solution repeated twice is counted as 2 solutions and so on.

Ex 1: Find the number of solutions or zeros for each equation or function.

a. $x^3 - 3x^2 + 9x - 27 = 0$

deg: 3
3 solutions

b. $x^4 + 6x^3 - 32x = 0$

deg: 4
4 solutions

Ex 2: Find all zeros of $f(x) = x^4 - 7x^3 + 13x^2 + x - 20$

p: $20 \rightarrow \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

q: $1 \rightarrow \pm 1$

$\frac{p}{q}$ = $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}$

$$\begin{array}{r|rrrrr} -1 & 1 & -7 & 13 & 1 & -20 \\ & & -1 & 8 & -21 & 20 \\ \hline & 1x^3 & -8x^2 & +21x & -20 & \boxed{0} \end{array}$$ ✓

$$\begin{array}{r|rrrr} 4 & 1 & -8 & 21 & -20 \\ & & 4 & -16 & 20 \\ \hline & 1x^2 & -4x & +5 & \boxed{0} \end{array}$$ ✓

$2 \pm i$

$\frac{4 \pm 2i}{2}$

$x^2 - 4x + 5$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} = \frac{4 \pm \sqrt{-4}}{2}$

$x = -1, 4, 2 \pm i$