**Learning Target(s):** I am able to use the Rational Zero Theorem to find all real zeros of a polynomial function.

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## Notes: 5.6 Find Rational Zeros

**The Rational Zero Theorem**

If $f(x) = a_n x^n + \ldots . a_1 x + a_0$ has integer coefficients, then every rational zero of f has the following form:

$\frac{p}{q} = \frac{\text{factor of constant}}{\text{factor of leading coefficient}}$ $\frac{a_0}{a_n}$

$5x+2$ no variable

**Ex 1:** Find all **real** zeros of $f(x) = x^3 - 4x^2 - 7x + 10$

**Step 1:** List all the possible rational zeros.

p: 10 → $\pm 5, \pm 2, \pm 10, \pm 1$

q: 1 → $\pm 1$

$\frac{p}{q} = \pm\frac{5}{1}, \pm\frac{2}{1}, \frac{\pm 10}{1}, \pm\frac{1}{1}$

**Step 2:** Test zeros using synthetic division (use a graphing calculator to help narrow down your possible zeros from step 1)

Y= Test 1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | -4 | -7 | 10 |
| | | 1 | -3 | -10 |
| | $1x^2$ | -3x | -10 | 0 ✓ |

**Step 3:** Factor the trinomial once you have reduced your original polynomial to a quadratic.

$x^2 - 3x - 10$

$(x-5)(x+2)$

$x-5=0$ $\quad$ $x+2=0$

$x=5$ $\quad$ $x=-2$

**Ex 2:** Find all **real** zeros of $f(x) = 8x^4 + 2x^3 - 21x^2 - 7x + 3$

**Step 1:**

$p: 3 \to \pm 1, \pm 3$

$q: 8 \to \pm 2, \pm 4, \pm 1, \pm 8$

$\frac{p}{q} = \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{1}, \pm\frac{1}{8}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{3}{1}, \pm\frac{3}{8}$

**Step 2:**

$\sqrt{-\frac{3}{2}}$ $-1.5$

2nd window

changed the $\Delta$Tbl 0.25

$-\frac{3}{2}$ | 8 2 -21 -7 3

↓ -12 15 9 -3

$8x^3 - 10x^2 - 6x + 2$ | 0 ✓

$8x^3 - 10x^2 - 6x + 2$

$\frac{1}{4}$ | 8 -10 -6 2

2 -2 -2

$8x^2 - 8x - 8$ | 0 ✓

$8x^2 - 8x - 8 = 0$

$\frac{8}{8}(x^2 - x - 1) = \frac{0}{8}$

$x^2 - x - 1 = 0$

$\frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} \to \frac{1 \pm \sqrt{5}}{2}$

zeros: $-\frac{3}{2}, \frac{1}{4}, \frac{1 \pm \sqrt{5}}{2}$ $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$