

Learning Target(s): I can divide polynomials using long division and/or synthetic division.
I can use the Factor Theorem and Remainder Theorem to solve polynomial functions.

Notes: 5.5 Apply the Remainder and Factor Theorems

Divide using long division: $38,452 \div 256$

$$\begin{array}{r}
 150 + \frac{52}{256} \\
 256 \overline{) 38,452} \\
 \underline{-256} \\
 1285 \\
 \underline{-1280} \\
 52
 \end{array}$$

150R 52

Remainder

Ex. 1 Divide $4x^4 + 5x^2 - 9x + 18$ by $x^2 + 2x + 4$ using long division.

$$\begin{array}{r}
 4x^2 - 8x + 5 + \frac{13x-2}{x^2+2x+4} \\
 x^2 + 2x + 4 \overline{) 4x^4 + 0x^3 + 5x^2 - 9x + 18} \\
 \underline{-4x^4 - 8x^3 + 16x^2} \\
 -8x^3 - 11x^2 - 9x \\
 \underline{+ 8x^3 + 16x^2 + 32x} \\
 5x^2 + 23x + 18 \\
 \underline{-5x^2 + 10x + 20} \\
 13x - 2
 \end{array}$$

Remainder

- ① Divide
- ② multiply
- ③ Subtract
- ④ Repeat

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $f(k)$.

Ex 2: Divide $f(x) = x^3 + 4x^2 - 5x + 3$ by $(x + 2)$ using synthetic division. take the opp. sign

$$\begin{array}{r}
 -2 \quad | \quad 1 \quad 4 \quad -5 \quad 3 \\
 \quad \quad | \quad \downarrow -2 \quad -4 \quad 18 \\
 \hline
 1x^2 + 2x - 9 \quad | \quad 21
 \end{array}$$

Remainder

$$x^2 + 2x - 9 + \frac{21}{x+2}$$

You can use the remainder theorem to check your answer:

$$\begin{aligned}
 f(-2) &= (-2)^3 + 4(-2)^2 - 5(-2) + 3 \\
 &= 21
 \end{aligned}$$

Factor Theorem – A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Ex 3: Factor $f(x) = 2x^3 - 11x^2 + 3x + 36$ completely given that $(x - 3)$ is a factor.

$\frac{x^3}{x}$

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2	-11	3	36
↓	6	-15	-36
2x ² - 5x - 12 0 ✓			

Factor → 2x² - 5x - 12

$(2x+3)(x-4)(x-3)$

Yes! it's a factor

2x	2x ²	-8x
3	3x	-12

-24

Ex 4: One zero of $f(x) = x^3 + 4x^2 - 15x - 18$ is $x = -1$. Find the other zeros.

If $x = -1$ is a zero then $(x + 1)$ is a factor of f .

$\frac{x^3}{x}$

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-1	1	4	-15	-18
		-1	-3	18
1x ² + 3x - 18 0 ✓				

Factor → x² + 3x - 18

$(x+6)(x-3)(x+1)$

$x+1=0$
 $x=-1$

$x-3=0$
 $x=3$

$x+6=0$
 $x=-6$

Try it!

1. Use both long division and synthetic division to divide $x^3 - 6x^2 + 9$ by $x - 4$.

$\frac{x^3}{x}$

$\frac{-2x^2}{x}$

$\frac{-8x}{x}$

x-4	x ³	-6x ²	+0x	+9
	-x ³	+4x ²		
	-2x ² +0x			
		+2x ² +8x		
		-8x+9		

$x^2 - 2x - 8 + \frac{-23}{x-4}$

-8x+9

+8x+32

-23

Remainder

2. Factor $f(x) = 3x^3 + 8x^2 + 3x - 2$ given that $x + 2$ is a factor.

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-2	3	8	3	-2
	↓	-6	-4	2
3 2 -1 0 ✓				

3x² + 2x - 1 → (3x-1)(x+1)(x+2)