Caveman Math

Counting Numbers (natural numbers) 1, 2, 3, …

Once upon a time, long, long ago, cavemen roamed the Earth. The men spent their days hunting, while the women spent their days gathering. One day Matilda grew bored as she gathered blueberries. She asked her bae Grok how many blueberries she needed to collect before she could stop. He grunted at her, and then said "many." "Well, how many is many?" she replied. He couldn't answer her. So she began counting to see how many she had collected. "One, two three… two hundred fifty-eight, two hundred fifty nine…" She counted all the way up to 312. "Well, that seems like many to me!" she thought to herself.

Whole numbers (we need ZERO) 0, 1, 2, 3, …

Well, it turns out they ate all of the blueberries in one night! After all the blueberries were gone, Grok said, "Matilda, give me some blueberries!" and Matilda replied, "I can't; there aren't any left!" To which Grok replied, "What do you mean there aren't ANY left? How many blueberries are there?" Matilda replied, "There are ZERO blueberries, you nitwit!!!"

Negative numbers (Integers) …-3, -2, -1, 0, 1, 2, 3, …

Grok was still quite hungry, so he decided to see if his friend Chewy had any blueberries. Luckily Chewy had an abundance of blueberries, and he was happy to give them to Grok, provided that Grok paid him back in blueberries the next week. Unfortunately, Matilda was not too happy about Grok borrowing the blueberries. "Now I have to collect even MORE blueberries because we OWE Chewy blueberries next week!" Matilda was very perturbed, and now that she knew how to count, she wanted to know how to express how many blueberries they currently had. Grok said, 'I dunno, I guess we have zero blueberries." "No, we do not!" replied Matilda. "We OWE blueberries so we have NEGATIVE 100 blueberries." "What in carnation does 'negative' mean?" said Grok. "You numbskull, the negative means we OWE Chewy blueberries!" replied Matilda, now getting very exasperated. The next day, Matilda gathered more blueberries, and she took them over to Chewy to pay him back.

Rational numbers (can be written as fractions)

The next day, Matilda was sick of eating plain old blueberries, so she decided to bake a pie (these were very advanced cavemen!) As she and Grok sat down to eat the pie, Chewy and his bae Gertrude arrived, ravenously hungry. Grok was not too happy that he had to share his pie not only with Matilda, but also with Chewy and Gertrude. "But how much pie am I going to get?" he whined. "Well, you'll get a piece of the pie," replied Gertrude. "But how much is that?" said Grok. Matilda thought for a long while. All of the numbers that she had been using so far didn't express how much Grok was going to get. She was dividing one pie by four people. "I've got it!" she exclaimed. "We're each going to get ¼ of the pie!" They all seemed satisfied with this answer, so they sat down and devoured the delicious blueberry pie.

THE END

Irrational Numbers:

We are not saying irrational means absurd, foolish, or silly. We're discussing irrational in the math context. This irrational means a number with lots of messy numbers after the decimal- a number that can't be written as a fraction- a number which is not a rational number.

What an unfortunate name for a number!

Years ago, negative numbers were actually called fictitious numbers. That was an unfortunate name also but is no longer used. All the numbers discussed thus far- counting, whole, integer, rational, and irrational- are subsets of the set of numbers called the REAL numbers. Yes, there are numbers which are not real.

Learning Target(s): I can solve a quadratic equation using complex numbers. I can add, subtract, multiply, and divide complex numbers. I can plot complex numbers in a coordinate plane.

Notes: 4.6 Perform Operations with Complex Numbers

Imaginary Unit i – $i = \sqrt{-1}$ so $\sqrt{-16} =$ $\sqrt{16} \cdot \sqrt{-1} = 4i$

The powers of i

$i =$ i $\sqrt{-1}$

$i^2 = i\,(i) =$ $\sqrt{-1}\cdot\sqrt{-1} = -1$

$i^3 = i^2\,(i) =$ $-i$

$i^4 = i^3\,(i) =$ $-i \cdot i = -i^2 = -(-1) = 1$

$i^5 = i^4\,(i) =$ i

$i^6 = i^5\,(i) =$ $i \cdot i = i^2 = -1$

warm up → $i^{213} =$

Examples:

1) Simplify $\sqrt{-9}$ $= \sqrt{9}\cdot\sqrt{-1}$

$= 3i$

2) Solve: $x^2 + 11 = 3$

$-11 \quad -11$

$\sqrt{x^2} = \pm\sqrt{-8} = \sqrt{8}\cdot\sqrt{-1}$

$2\sqrt{2}\cdot i$

$\boxed{x = \pm 2i\sqrt{2}}$

3) Solve: $2x^2 + 15 = -35$

$-15 \quad -15$

$\frac{2x^2}{2} = \frac{-50}{2}$

$\sqrt{x^2} = \sqrt{-25}$

$\sqrt{25}\cdot\sqrt{-1}$

$\boxed{x = \pm 5i}$

4) Solve: $3x^2 + 13 = -23$

$-13 \quad -13$

$\frac{3x^2}{3} = \frac{-36}{3}$

$\sqrt{x^2} = \sqrt{-12}$

$\boxed{x = \pm 2i\sqrt{3}}$

Complex Numbers in Standard Form: **a + bi (the real part + imaginary part)**

Adding and Subtracting Complex Numbers: Complex numbers can be added and subtracted by combining real parts together and combining the imaginary parts together. *Think like terms.*

Examples: (rewrite without parentheses first)

5) $(8 + 2i) + (5 + 4i) =$

$8 \boxed{+2i} + 5 \boxed{+4i}$

$13 + 6i$

6) $(7 - 6i) - (4 - 10i)$

distribute –

$7 \boxed{-6i} - 4 \boxed{+10i}$

$3 + 4i$

7) $3i + (-4 - 8i)$

$\boxed{3i} + (-4) \boxed{-8i}$

$-4 - 5i$

8) $(-2 - 6i) - (3 + 9i)$

$-2 \boxed{-6i} - 3 \boxed{-9i}$

$-5 - 15i$

Multiplying Complex Numbers: Complex numbers can be multiplied. Just remember that $i^2 = -1$.

Examples: (rewrite without parentheses first)

9) $6(2i - 3)$

$12i - 18 = -18 + 12i$

10) $-2i(i - 7)$

$-2i^2 + 14i$

$-2 \cdot -1$

$\boxed{2 + 14i}$

11) $(6 - 3i)(2 + 4i)$

$12 + 24i - 6i - 12i^2$

$-12 \cdot -1$

**When you multiply binomials you must FOIL and replace i^2 with -1.

$\boxed{12} + 18i \boxed{+12}$ $24 + 18i$

Complex Conjugates: a+bi and a−bi
(to write the conjugate, change ***only*** the sign in between the two terms)

Examples: Write the complex conjugate of each.

12) 3 – 2i: 3+2i 13) -2 + 4i: −2−4i 14) -5 – i: −5+i

Dividing Complex Numbers: To be simplified, complex numbers should not have any imaginary parts in the denominator (*think radicals*). Multiply numerator and denominator by the complex conjugate. Just remember that $i^2 = -1$.

Examples: (rewrite without parentheses first)

conjugate 1+4i

15) $\dfrac{7+5i}{1-4i} \cdot \dfrac{(1+4i)}{(1+4i)} = \dfrac{\boxed{7} + 28i + 5i + 20i^2}{\boxed{1} + \cancel{4i} - \cancel{4i} - 16i^2}$

$33i$ $\boxed{-20}$ $\boxed{+16}$

$$\boxed{\dfrac{-13 + 33i}{17}}$$

16) $\dfrac{6+4i}{2+i} \cdot \dfrac{(2-i)}{(2-i)} = \dfrac{12 - 6i + 8i - 4i^2}{4 - \cancel{2i} + \cancel{2i} - i^2}$

$+2i$ $+4$ $+1$

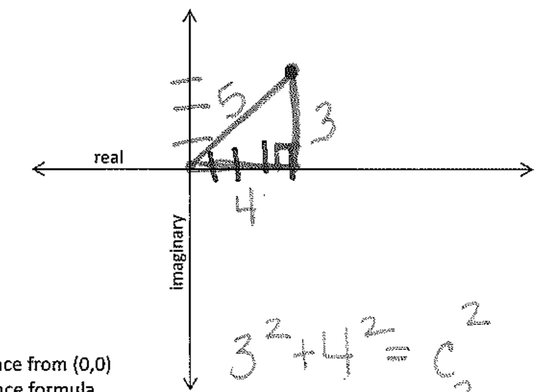
$$\boxed{\dfrac{16 + 2i}{5}}$$

I. **Graphing Complex Numbers**
When plotting complex numbers, plot the real part on the x-axis and the imaginary on the y-axis.

x y

16) Plot (4 + 3i)

real imaginary



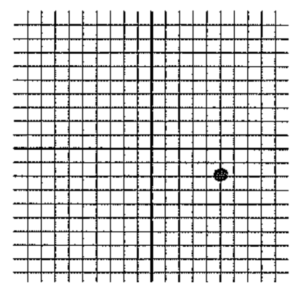
Absolute Value of a complex number (distance from (0,0)
To find the distance from zero, use the distance formula or draw a triangle and use the Pythagorean Theorem.

$|4 + 3i|$ 5

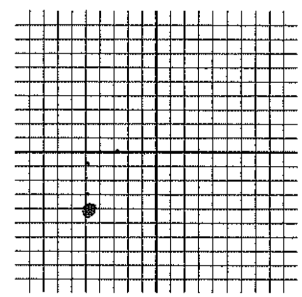
$3^2 + 4^2 = c^2$
$9 + 16 = c^2$
$25 = c^2$
$5 = c$

Ex 17: Plot the complex numbers in the same complex plane.

a. $5 - 2i$



b. $-5 - 4i$



Ex 18: Find the absolute value of each

a. $|5 - 12i|$

$5^2 + (-12)^2 = c^2$
$25 + 144 = c^2$
$169 = c^2$
$13 = c$

b. $|-6i|$

$6^2 = c^2$
$6 = c$